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Cosmology with dark energy decaying through its chemical-potential contribution

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Abstract
The consideration of dark energy's quanta, required also by thermodynamics, introduces its chemical potential into the cosmological equations. Isolating its main contribution, we obtain solutions with dark energy decaying to matter or radiation. When dominant, their energy densities tend asymptotically to a constant ratio, explaining today's dark-energy–dark-matter coincidence, and in agreement with supernova redshift data.

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1. Introduction

Dark energy is a component of the universe whose negative pressure, characteristic of the quantum vacuum, accelerates its expansion. Evidence for its existence has recently accumulated from independent sources such as the supernova redshift far-distance relation [1, 2], structure formation [3], the microwave background radiation [4] and lensing [5].

The cosmological constant Λ, dark energy's original conception, was added by Einstein in the application of general relativity to cosmology in 1917 in order to describe a static universe [6], building on a 1890s proposal by Neumann and Seeliger, who introduced it in a Newtonian framework for the same reasons. Its contribution in the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{1}$$

equilibrates gravity's attraction in a matter universe; here $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, which describe the geometry, and $T_{\mu\nu}$ is the energy-momentum tensor; we use units with the Newton, Planck, Boltzmann and light-speed constants $G = \hbar = k_B = c = 1$, except when given explicitly, as needed.

Zeld'ovich sought to connect it to the quantum vacuum [7]. This requires its reinterpretation as a $T_{\mu\nu}$ component in equation (1). The vacuum energy density of particle fields with mass $m \ll M_P = \frac{1}{\sqrt{G}}$ is obtained by summing over its modes $\mathbf{k}$:

$$\rho_{\Lambda P} = \frac{1}{(2\pi)^3} \int^{M_P} \mathrm{d}^3 k \sqrt{k^2 + m^2} \simeq 3 \times 10^{114} \text{ GeV cm}^{-3}; \tag{2}$$

the natural cutoff is the Planck-mass scale M_P, the only possible mass conformed of $G, \hbar$, and c, while in today's universe $\rho_{\Lambda 0} \simeq 4 \times 10^{-6}$ GeV cm^{-3}. $\rho_{\Lambda 0}$ represents $\Omega_{\Lambda 0} = \rho_{\Lambda 0}/\rho_{c0} \simeq 0.73$ of its critical energy density ρ_{c0} today [8], and in a flat universe [9] $\sum \Omega_i = 1$. The rest corresponds mainly to matter, dark and baryonic, the latter conforming $\Omega_{b0} \simeq 0.044$ only [8]. Dark energy's origin, its smallness by 122 orders of magnitude with respect to the vacuum's natural Planck scale, and the coincidence of its present energy-density scale with that of the universe remain puzzling; dynamic behaviour points to a possible explanation.

The energy components are generally perfect fluids, described by their energy tensor $T_{\nu}^{\mu(i)} = (\rho_i, p_i, p_i, p_i)$ (at rest), with $T_{\mu\nu} = \sum_i T_{\mu\nu}^{(i)}$. Radiation and matter are characterized by an equation of state

$$p_i = w_i \rho_i, \tag{3}$$

where $w_r = 1/3$ for radiation (and for relativistic Fermi or Bose gases,) and $w_m = 0$ for non-relativistic matter. Under the isotropic Robertson–Walker metric $\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$, equation (1) implies the Friedmann equation

$$H^2 = \frac{8\pi}{3}\rho_c = \frac{8\pi}{3}(\rho_\Lambda + \rho_r + \rho_m), \tag{4}$$

where x, y, z are co-moving Cartesian coordinates, R is the scale factor, depending on time t, as do ρ_i, and $H = \dot{R}/R$, the Hubble parameter (a dot denotes time derivative.)

The energy-conservation equation within an expanding volume $V \sim R^3$,

$$\sum_i d(\rho_i V) = -\sum_i p_i \, \mathrm{d}V \tag{5}$$

is implied by the contraction of equation (1). When decoupled, each contribution also satisfies

$$\mathrm{d}(\rho_i V) = -p_i \, \mathrm{d}V. \tag{6}$$

Equation (6) can also be interpreted as a particular case of the first law of thermodynamics:

$$\mathrm{d}(\rho V) = -p \, \mathrm{d}V + \mu \, \mathrm{d}N + T \, \mathrm{d}S, \tag{7}$$

with additional contributions from the entropy S, and the particle number N, where T is the temperature and μ is the chemical potential. When non-interactive radiation has $\mu = 0$, baryonic matter is conserved, $\mathrm{d}N = 0$, and for both $\mathrm{d}S = 0$. These conditions may not be true for dark energy or dark matter. In this paper, we show that the consideration of dark energy's quanta modifies the cosmological equations through the $\mu \, \mathrm{d}N$ term in equation (7), with the implication that dark energy decays to another component. Thus, the derived asymptotic energy-density constant ratio of the dominant components reproduces the coincidence of dark energy and dark matter today. The entropy term $T \, \mathrm{d}S$ in equation (7) will be neglected, as dark energy is associated with low-energy states. We first classify the chemical potential associated with the pressure in equation (3) (section 2.) Relating it to a decay width, we consider its main contribution to the cosmological equations, which are exactly solved for two components, and then we apply the model to the supernova data (section 3.) We finally draw conclusions (section 4.)

2. Dark energy's equation of state

The form of equation (2) implies that Λ generates a pressure $p_\Lambda = -\rho_\Lambda$, so $w_\Lambda = -1$ for the vacuum energy. The parametric extension to arbitrary negative values w_Λ, following equation (3), with similar properties [10, 11], suits the lack of precise knowledge about it. Whatever is its nature, and with a name not bound to its constancy, dark energy should contain quanta [7], as any other form of energy in the universe, and so, the energy dependence on its number N should be accounted for. Within the relation

$$E = cV^{-w}, \tag{8}$$

consistent with equation (3), c being a constant, if the energy dependence remains extensive, another such quantity is required. Using N for such a variable,

$$E = c'N\left(\frac{V}{N}\right)^{-w} \tag{9}$$

introduces an N-dependence, with c' being an (intensive) constant, except in the $w = -1$ case, in agreement with the view that no quanta are associated with the vacuum.

Equation (9), also consistent with equation (3), implies the contribution

$$n\mu = (1 + w)\rho, \tag{10}$$

where $n = N/V$ is the particle density.

We concentrate on dark energy satisfying equation (3). Using the thermodynamic relation

$$s = \frac{1}{T}(\rho + p - n\mu), \tag{11}$$

with $s = S/V$ the entropy density, we identify two limiting cases: (*1*) in the zero-entropy regime ($s = 0$),

$$\rho_{\Lambda w} = c_w n^{1+w}, \tag{12}$$

with c_w being a constant, and $n\mu_{\Lambda w} = (1 + w)\rho$, as for equation (10); (*2*) the radiation-like assumption, $\mu_{rw} = 0$, leads to

$$s_{rw} = c_{rw}\rho^{\frac{1}{1+w}} \tag{13}$$

(c_{rw} a constant.)

Case (*1*) with equation (10), induced from equation (3), or case (*2*) with $\mu_{rw} = 0$ represent special conditions; similarly to equation (3), the most general linear ρ-dependence for the chemical potential requires the new parameter χ in

$$n\mu_{w\chi} = (1 + w + \chi)\rho. \tag{14}$$

Equations (3), (11) and (14) then generally lead to $s_{w\chi} = n\left(\frac{\rho}{c_w n^{1+w}}\right)^{-\frac{1}{\chi}}$. From the resulting temperature $T_{w\chi} = -\frac{\chi\rho}{n}\left(\frac{\rho}{c_w n^{1+w}}\right)^{\frac{1}{\chi}}$, it follows that $\chi \neq 0$ signals a non-zero $T_{w\chi}$. In fact, $s_{w\chi}$ contains the $s = 0$ limit, as equation (12) is approached with $\rho \sim \rho_{\Lambda w}$ for $\chi \to 0$, and for the $\mu_{rw} = 0$ case in equation (13), $s_{w\chi} = s_{rw}$ for $\chi = -w - 1$, and $c_{rw} = c_w^{1/\chi}$. The knowledge of w_Λ, and these limits suggest χ is also O(1).

The modification of equation (6) by the chemical-potential contribution is analysed next.

3. Cosmological equations with dark energy's chemical potential

The chemical potential can be written as

$$\mu_\Lambda \, \mathrm{d}N = \mu_\Lambda (n_\Lambda \, \mathrm{d}V + V \, \mathrm{d}n_\Lambda); \tag{15}$$

changes in particle numbers through decay are associated with partial widths Γ, and, ultimately, with interactions. In the universe's evolution in $\mathrm{d}t$, we distinguish the two contributions: (*1*) $N\Gamma_1 \, \mathrm{d}t = n_\Lambda \mu_\Lambda \, \mathrm{d}V = (1 + w_\Lambda + \chi)\rho_\Lambda \, \mathrm{d}V$ is associated with decay due to its expansion

$$n_\Lambda \Gamma_1 = 3(1 + w_\Lambda + \chi) H \rho_\Lambda \sim \rho_\Lambda^{3/2}, \tag{16}$$

given $H \sim \rho_\Lambda^{1/2}$; (*2*) $N\Gamma_2 \, \mathrm{d}t = \mu_\Lambda V \, \mathrm{d}n_\Lambda$ contains terms that are not of this form; it could account for any other out-of-equilibrium conceivable decay process linked to interactions. For the gravitational interaction, and $T_{w\chi} = 0, \Gamma_2 \sim \sigma n_\Lambda v \sim \left(1/M_P^4\right) n_\Lambda \rho_\Lambda^{1/2}$, where for the cross section $\sigma \sim \left(1/M_P^4\right)\rho_\Lambda^{1/2}$, given a tree-level gravitational interaction, and the dimensionally fit power of the only relevant variable ρ_Λ; the velocity $v \sim c = 1$, so $n_\Lambda \Gamma_2 \sim \rho_\Lambda^{\frac{2}{w_\Lambda+1}+1/2}$, using $\rho_{\Lambda w}$ in equation (12). Comparing with $n_\Lambda \Gamma_1 \sim \rho_\Lambda^{3/2}$, from equation (16), for $-1 < w_\Lambda < 1, \Gamma_2 \ll \Gamma_1$ as $\rho_\Lambda \to 0$. Similarly, this will always occur for low $T_{w\chi} \neq 0$, implying still $\rho_\Lambda \sim \rho_{\Lambda w}$, but high enough for the thermic contribution to be dominant so [12] $\sigma \sim \left(1/M_P^4\right) T_{w\chi}^2$. Another type of interaction can be dominant for some time, but it will eventually be overridden by the Γ_1 term. Lower powers of ρ_Λ, e. g., a constant decay rate $n_\Lambda \Gamma_2 \sim \rho_\Lambda$, could make a significant cosmological contribution, but it would have to be fine tuned to give the present parameters [13]. Thus, the Γ_2 term can and will be neglected.

Under such circumstances, we use changes of the form $\partial N_\Lambda / \partial V = n_\Lambda$ in equation (15). We obtain, using equations (7), (16),

$$\dot{\rho}_\Lambda + 3(w_\Lambda + 1) H \rho_\Lambda = 3[(w_\Lambda + 1) + \chi] H \rho_\Lambda. \tag{17}$$

Energy conservation in equation (5) demands that energy be transferred, which we assume occurs for only another dominant *i* component in equation (4),

$$\dot{\rho}_i + 3(w_i + 1) H \rho_i = -3[(w_\Lambda + 1) + \chi] H \rho_\Lambda. \tag{18}$$

The set of equations (4), (17), (18) describes a two-fluid system with ρ_Λ decaying out of equilibrium as is common in many universe processes [12]. No energy transfer is produced for $w_\Lambda + 1 + \chi = 0$, that is, for the radiation-like case with $n\mu_{w\chi} = 0$ in equation (14). We also find (see equation (17)) dark-energy decay for $\chi < 0$. A decaying cosmological constant was first conceived by Bronstein [14] to explain the universe's time direction, and recent study starts with [15], with various phenomenological decay laws then considered [16]; quintessence models with a similar energy interchange have also been studied [17]. By substituting H in equation (4) into equation (17), we obtain

$$\rho_i = -\rho_\Lambda + \frac{\dot{\rho}_\Lambda^2}{24\pi \chi^2 \rho_\Lambda^2}. \tag{19}$$

Substituting this into equation (18), we get

$$6\chi \rho_\Lambda \ddot{\rho}_\Lambda + (d_i - 6\chi)\, \dot{\rho}_\Lambda^2 - 24\pi [d_i - 3\,(1 + w_\Lambda)] \chi^2 {\rho_\Lambda}^3 = 0, \tag{20}$$

where $d_i = 3(w_i + 1)$. t as an inverse function of ρ_Λ can be integrated, where initially ρ_{Λ_i} at t_i,

$$t - t_i = \int_{\rho_\Lambda}^{\rho_{\Lambda_i}} \mathrm{d}\rho \left(\frac{d_i + 3\chi}{24\chi^2 \pi [d_i - 3(w_\Lambda + 1)]\rho^3 + 3(d_i + 3\chi)\chi C \rho^{2 - \frac{d_i}{3\chi}}} \right)^{\frac{1}{2}}. \tag{21}$$

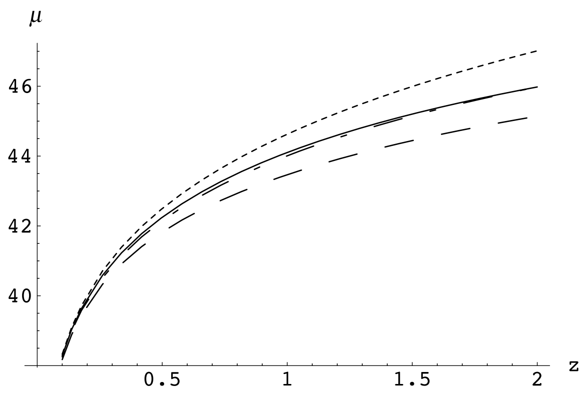

Figure 1. Comparison of magnitude $\mu = 5\mathrm{Log}_{10}(d_L/\mathrm{Mpc}) + 25$ of luminosity distance d_L, as a function of redshift z, for flat models. For non-asymptotic models with $w_\Lambda = -1$, and (*a*) $\Omega_{m0} = 0$, $\Omega_{\Lambda 0} = 1$ (dotted), (*b*) $\Omega_{m0} = 0.27$, $\Omega_{\Lambda 0} = 0.73$ (line), and (*c*) $\Omega_{m0} = 1$, $\Omega_{\Lambda 0} = 0$ (dashed); and (*d*) for asymptotic model with $\Omega_{b0} = 0.044$, and $\chi_0 = -0.48$ (dot-dashed). The reduced Hubble parameter $h = 0.71$ was used for all cases.

C accounts for initial conditions for ρ_i, and we have chosen the solution for which R increases and ρ_Λ decreases. For some χ, w_Λ, $t(\rho_\Lambda)$ can be given explicitly in terms of hypergeometric and elliptic functions. Using equations (19), (21) one finds

$$\rho_c \approx \frac{24\chi^2\pi[d_i - 3(w_\Lambda + 1)]\rho_\Lambda + (d_i + 3\chi)3\chi C\rho_\Lambda^{-\frac{d_i}{3\chi}}}{24\pi\chi^2(d_i + 3\chi)}. \tag{22}$$

One derives that for $-d_i/3 < \chi < 0$

$$\lim_{\rho_\Lambda \to 0} \frac{\rho_\Lambda}{\rho_c} = \frac{d_i + 3\chi}{d_i - 3(w_\Lambda + 1)} \tag{23}$$

within the wide set of initial conditions $C \ll \rho_{\Lambda 0}^{1+\frac{d_i}{3\chi}}$, so Ω_i and Ω_Λ will acquire a fixed asymptotic value.

Such an asymptotic behaviour fits the supernova data [18] interpreted under equation (23), with dark matter and dark energy evolving with a constant ratio. Considering baryonic matter, dark matter and dark energy (the latter two evolving as $R^{3\chi_0}$), assuming asymptotic behaviour sets in as early as $z = 2$, with the constant $\chi_0 = -0.48$, and as shown in figure 1 (and compared with the fitting non-asymptotic model, and non-fitting $\Omega_{\Lambda 0} = 0$, and the cosmological constant $\Omega_{\Lambda 0} = 1$ cases), one can reproduce the luminosity distance $d_L = H_0^{-1}(1 + z)\int_0^z \mathrm{d}z'[\Omega_{b0}(1 + z')^3 + (1 - \Omega_{b0})(1 + z')^{-3\chi_0}]^{-1/2}$ up to the measured redshift $z \sim 2$. We note that the fit is independent of $\Omega_{\Lambda 0}$, as derives from the asymptotic regime. The choice of initial conditions (C in equation (21)) sets the timing of the matter-dominated regime ($w_i = w_m = 0$ in equation (18)) before the asymptotic one, to match the conventional cosmology.

4. Conclusions

In summary, the account of dark energy's quanta allows for a dark-energy decaying model able to explain its coincidence with dark matter today, within classical general relativity and thermodynamics. It represents a departure from the zero-temperature cosmological constant, while it maintains the results of the standard cosmology. This supports a conservative approach in which known physical elements can provide new information [19]. Dark energy's coincidence with the critical density today is connected to the universe evolution, in which events occur by contingency, rather than chance. While microphysics [20] needs to elucidate the dark energy's equation of state, the universe already emerges as flat, interconnected, evolving deterministically, and in an inexorable process of accelerated expansion and decay.

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